CENTRAL UNIVERSITY OF HARYANA

Term End Examination January 2023

Programme : **M.Sc. Statistics** **Session: 2022-23**

Semester : **Third**

Course Title : **Econometrics** *Max. Time: 3 Hours*

Course Code : **SBS ST 01 303 C 3104** *Max. Marks : 70*

Instructions:

1. Question no. 1 has seven parts and students need to answer any four. Each part carries three and half Marks.
2. Question no. 2 to 5 have three parts and student need to answer any two parts of each question. Each part carries seven marks.

Question No. 1. **(4x3.5=14)**

a. Obtain ordinary least squares estimators for the parameters in case of simple linear regression model and discuss their properties.

b. Derive confidence interval estimate for the slope parameter in simple linear regression model.

c. Describe the coefficient of determination in simple linear regression model.

d. Explain the assumption used in multiple linear regression model.

e. Discuss the generalized least square estimation method for the multiple linear regression model.

f. Prove that the restricted regression estimator $\hat{\underset{\sim}{\beta}}$ is an unbiased estimator of $\underset{\sim}{\beta}$.

g. Give sources of multicollinearity in regression analysis.

Note: Question number **Two to Five** have three sub parts and students need to answer **any two-sub part** of each question. Each sub part carries **seven marks.**

Question No. 2 **(2x7=14)**

a. Prove that the generalized least squares estimator (GLSE) $\hat{\underset{\sim}{\beta}}$ is the best linear unbiased estimator (BLUE) of $\underset{\sim}{\beta}$.

b. Obtain restricted regression estimator of $\underset{\sim}{\beta}$ in multiple linear regression model $\underset{\sim}{y} = \underset{\sim}{X}\underset{\sim}{\beta} + \underset{\sim}{\varepsilon}$.

c. Describe estimation under the first order autoregressive process and give consequences of auto correlated disturbances.

Question No. 3 **(2x7=14)**

a. Obtain ridge regression estimator for the parameters in multiple linear regression model. Also, discuss its properties.

b. What do you mean by heteroskedasticity? Discuss any one test for heteroskedasticity.

c. Discuss the consequences of multicollinearity.

Question No. 4 **(2x7=14)**

a. In multiple linear regression model $\underset{\sim}{y} = \underset{\sim}{X}\underset{\sim}{\beta} + \underset{\sim}{\varepsilon}$, obtain instrumental variable estimator of $\underset{\sim}{\beta}$ and show that it is a consistent estimator.

b. Discuss difference between disturbances and measurement errors with an example?

c. Discuss the Wald's method in measurement error models.

Question No. 5 **(2x7=14)**

a. Discuss the indirect least squares estimation method.

b. Discuss the identification problem in simultaneous equations models.

c. Discuss the two stage least squares estimation method.

CENTRAL UNIVERSITY OF HARYANA

Term End Examinations, January 2023

Programme: M.Sc (Statistics) **Session: 2021-22**

Semester: III **Max. Time: 3 Hours**

Course Title: Statistical Inference-II **Max. Marks: 70**

Course Code: SBS ST 01 302 C 3104

Instructions:

1. Question no. 1 has seven sub parts and students need to answer any four. Each sub part carries three and half Marks.
2. Question no. 2 to 5 have three sub parts and students need to answer any two sub parts of each question. Each sub part carries seven marks.

Question No. 1. (4X3.5=14)

a) Explain the following terms with suitable example (i) uniform prior, and (ii) noninformative prior.

b) What is squared error loss function? Give a situation where it is applicable.

c) Differentiate between frequentist and Bayesian inference approach.

d) Explain what is meant by non-parametric methods. How do they differ from parametric methods?

e) Describe the credible and highest posterior density (HPD) credible interval.

f) Define the following terms with suitable example (i) Kernel (ii) distribution free method.

g) Explain how the sequential test procedure differs from the Neyman-Pearson test procedure.

Question No. 2. (2X7=14)

a) Show that noninformative priors are scale invariant. Derive the asymptotically locally invariant prior for exponential family of distributions.

b) Explain approaches for the subjective determination of prior density with appropriate example.

c) Obtained Bayes estimators of μ and σ^2 under symmetric loss function in the lognormal probability density function $f(x|\mu,\sigma)=\frac{x^{-1}}{\sqrt{2\pi}\sigma}\exp\left[-\frac{1}{2\sigma^2}(\log x-\mu)^2\right]\ \infty<\mu<\infty,\sigma,x>0$ when the joint prior is $g(\mu,\sigma)\propto\frac{1}{\sigma^c},\ c>0.$

Question No. 3. (2X7=14)

a) Explain the prior and posterior analysis of exponential distribution. Find the Bayes estimator of parameter in exponential distribution. Show that under absolute error loss, the Bayes rule is the median of the posterior distribution.

b) Let $\underline{x}=(x_1,x_2,...,x_n)$ be a random sample from the exponential probability density function $f(x\mid\mu,\theta)=\frac{1}{\theta}\exp\left(-\frac{x-\mu}{\theta}\right)$, $0<\mu<x,\theta>0$, obtain $(1-\alpha)\%$ credible and highest posterior density (HPD) credible intervals for μ and θ under the vague-prior $g(\mu,\theta)\propto\frac{1}{\theta}$, $\theta>0$.

c) If a sufficient statistic exists for the parameter p of binomial distribution $B(n,p)$ then show that family of conjugate priors exists for p.

Question No. 4. (2X7=14)

a) Derive the Wald-Wolfowitz run test for testing the equality of two distribution functions.

b) Write short notes on the following:

(i) Empirical distribution function

(ii) K-S test

c) Describe the median test for the two-sample location problem. Find the distribution of the test statistic and compute its mean and variance under the null hypothesis. How is the test carried out in case of large samples?

Question No. 5. (2X7=14)

a) Describe Wald's Sequential Probability Ratio test. Define the OC function and ASN function in sequential analysis.

b) Let X have the distribution $f(x,\theta)=\theta^x(1-\theta)^{1-x}$, $x=0,1;\ 0<\theta<1$. For testing $H_0:\theta=\theta_0$ against $H_1:\theta=\theta_1$, construct S.P.R.T. and obtain its A.S.N. and O.C. functions.

c) Develop the S.P.R.T. for testing $H_0:\theta=\theta_0$ against $H_1:\theta=\theta_1\ (>\theta_1)$, based on a random sample of size n from a population with pdf $f(x,\theta)=\frac{1}{\theta}e^{-x/\theta}$, $x>0$, $\theta>0$. Also obtain it's A.S.N. and O.C. functions.

CENTRAL UNIVERSITY OF HARYANA

Term End Examinations, January 2023

Programme:	**M.Sc. Statistics**	**Session: 2022-23**
Semester:	**III**	**Max. Time: 3 Hours**
Course Title:	**Multivariate Analysis**	**Max. Marks: 70**
Course Code:	**SBS ST 01 301 C 3104**	

Instructions:

1. Question no. 1 has seven sub parts and students need to answer any four. Each sub part carries three and half Marks.

2. Question no. 2 to 5 have three sub parts and students need to answer any two sub parts of each question. Each sub part carries seven marks.

Question No. 1. **(4X3.5=14)**

a) Define Hotelling's T^2-statistic and prove that it is invariant under a non-singular linear transformation.

b) Let $\underline{X}_\alpha$ $(\alpha = 1,2\ldots,p)$ be the random sample from $N_p(\underline{\mu},\Sigma)$, where Σ is known.

Develop a suitable test for testing $H_0 : \underline{\mu} = \underline{\mu}_0$ where $\underline{\mu}_0$ is given vector.

c) If $A \sim W_p(n,\Sigma)$, then prove that $DAD' \sim W_p(n, D\Sigma D')$, where D is a $p \times p$ non-singular matrix of constant elements.

d) If the variance-covariance matrix of p-component random vector $\underline{X}$ is Σ, then obtain the variance-covariance matrix of the transformed vector $\underline{Y} = A\underline{X}$, where A is $p \times p$ matrix of constant elements.

e) Let X_1 and X_2 are jointly distributed normal variates with $E(X_i) = 0$ and $\text{Var.}(X_i) = 1$, for $i = 1,2$. If the distribution of X_2 given X_1 is normal $N_1(0, 1-\rho^2)$ with $|\rho| < 1$, find the covariance matrix of X_1 and X_2.

f) Let $\underline{X} \sim N_3(\underline{0},\Sigma)$, where $\Sigma = \begin{pmatrix} 1 & \rho & 0 \\ \rho & 1 & \rho \\ 0 & \rho & 1 \end{pmatrix}$ find ρ such that $X_1 + X_2 + X_3$ and $X_1 - X_2 - X_3$ are independent.

g) If $X_1, X_2, \ldots, X_p$ drawn from $N_p(\underline{\mu},\Sigma)$ distribution. Then find the distribution of $(\underline{X} - \underline{\mu})'\Sigma^{-1}(\underline{X} - \underline{\mu})$.

Question No. 2. **(2X7=14)**

a) Define Wishart distribution. Show that Wishart distribution is a generalization of the chi-square distribution.

b) Let $\underset{\sim}{X}_\alpha$ $(\alpha = 1,2\ldots,n)$ be the random sample of size n from $N_p(\underset{\sim}{\mu}_0, \Sigma)$, where $\underset{\sim}{\mu}_0$ is a given vector. Derive the maximum likelihood estimator of Σ.

c) If X_i $(i = 1,2)$ are distributed independently according to $W_p(\upsilon_i, \Sigma)$, respectively, then show that $X_1 + X_2 \sim W_p(\upsilon_1 + \upsilon_2, \Sigma)$.

Question No. 3. **(2X7=14)**

a) Let $\underset{\sim}{X}_1, \underset{\sim}{X}_2, \underset{\sim}{X}_3$ and $\underset{\sim}{X}_4$ be independent $N_p(\underset{\sim}{\mu}, \Sigma)$ random vectors, if $\underset{\sim}{U}_1 = \frac{1}{4}\underset{\sim}{X}_1 - \frac{1}{4}\underset{\sim}{X}_2 + \frac{1}{4}\underset{\sim}{X}_3 - \frac{1}{4}\underset{\sim}{X}_4$, and $\underset{\sim}{U}_2 = \frac{1}{4}\underset{\sim}{X}_1 + \frac{1}{4}\underset{\sim}{X}_2 - \frac{1}{4}\underset{\sim}{X}_3 - \frac{1}{4}\underset{\sim}{X}_4$, find the joint distribution of $\underset{\sim}{U}_1$, and $\underset{\sim}{U}_2$, and their marginals.

b) What do you mean by generalized variance obtain the distribution of sample generalized variance.

c) Show that multiple correlation is invariant under the non-singular linear transformation.

Question No. 4. **(2X7=14)**

a) Explain the Likelihood ratio criteria for testing $H_0 : \rho_{1(2,3,\ldots,p)} = 0$.

b) Let $\underset{\sim}{X} \sim N(\underset{\sim}{\mu}, \Sigma)$, where $\underset{\sim}{X}' = (X_1, X_2, X_3)$, $\underset{\sim}{\mu}' = (0,0,0)$, and

$$\Sigma = \begin{pmatrix} 7 & 3 & 2 \\ 3 & 4 & 1 \\ 2 & 1 & 2 \end{pmatrix}.$$ Obtain the distribution of $X_1 + 2X_2 - 3X_3$.

c) Derive the sampling distribution of sample correlation coefficient r on the basis of a random sample of size n from a bivariate normal distribution $N_2(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$, if $\rho = 0$. Also find $E(r^k)$.

Question No. 5. **(2X7=14)**

a) Define canonical correlation and canonical variates. Prove that canonical correlations are invariants under a non-singular linear transformation.

b) Prove that multiple correlation is a particular case of canonical correlation.

c) Define principal component analysis. Let $$\Sigma = \sigma^2 \begin{pmatrix} 1 & \rho & \cdots & \rho \\ \rho & 1 & \cdots & \rho \\ \vdots & \vdots & \vdots & \vdots \\ \rho & \rho & \cdots & 1 \end{pmatrix}.$$ Find the first principal component.

CENTRAL UNIVERSITY OF HARYANA

Second Semester Term End Examinations January 2023

Programme : M.Sc. Statistics

Semester : Third

Course Title : Stochastic Processes

Course Code : SBS ST 01 301 DCE 3104

Session: 2022-23

Max. Time: 3 Hours

Max. Marks: 70

Instructions:

1. Question no. 1 has seven parts and students need to answer any four. Each part carries three and half Marks.
2. Question no. 2 to 5 have three parts and student need to answer any two parts of each question. Each part carries seven marks.

Question No. 1.

(4x3.5=14)

a. Define transient, persistent and periodic recurrent events.

b. Show that the probability generating function for the binomial distribution $p_j = \binom{n}{j} p^j q^{n-j}$ is $(q+px)^n$. Also find its mean and variance.

c. Define renewal process with the help of suitable example.

d. Consider the three state Markov chain having transition probability matrix

$$\begin{bmatrix} 0 & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

. Find the stationary probability distribution.

e. Show that for a Galton Watson branching process, for r, n = 0,1, 2, ….

$$E\{X_{n+r}|X_n\} = X_n m^r$$

f. Define Absorption probabilities. The state transition matrix of the Markov chain is given by the following matrix:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & \frac{2}{3} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Let's define a_i as the absorption probability in state 3 if we start from state i. Obtain a_i for i= 0, 1, 2, 3.

g. Show that sum of two independent Poisson processes is a Poisson process.

Note: Question number **Two to Five** have three sub parts and students need to answer **any two sub part** of each question. Each sub part carries **seven marks.**

Question No. 2

(2x7=14)

a. Prove that the state j is persistent if the series $\sum_n p_{jj}^{(n)}$ is divergent and is transient if $\sum_n p_{jj}^{(n)}$ is convergent.

b. Define Stochastic Process. Discuss the types of stochastic process with the help of suitable examples.

c. Let $\{X_n, n \geq 0\}$ be a Markov chain having state space S={1, 2, 3, 4} and transition probability matrix

$$\begin{bmatrix} \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Show that

(1) State 3 and 4 is transient.

(2) State 1 is non null persistent.

Question No. 3 **(2x7=14)**

a. For a Galton Watson process with m=1 and $\sigma^2 < \infty$, we have

$\lim_{n\to\infty} \frac{1}{n}\left(\frac{1}{1-P_n(s)} - \frac{1}{1-s}\right) \to \frac{\sigma^2}{2}$. Uniformly in $0 \leq s < 1$.

b. In a classical gambling problem find the probability of ruin at the nth trial.

c. Prove that if m=1, the probability of ultimate extinction is 1. If m > 1 the probability of ultimate extinction is the positive root less that unity of the equation p(x) = x.

Question No. 4 **(2x7=14)**

a. Obtain the probability generating of the birth and death process with the effect of immigration.

b. Define simple death process and also obtain its probability generating function.

c. Discuss in detail the generalization of the Poisson process.

Question No. 5 **(2x7=14)**

a. Define Semi Markov process also prove that with probability 1.

$\frac{N(t)}{t} \to \frac{1}{\mu}$ as $t \to \infty$, where $\mu = E(X_n) \leq \infty$

b. State and Prove the Central Limit Theorem for Renewals.

c. If we have

$Pr(Y(t) \leq x) = F(t-x) - \int_0^t [1 - F(t+x-y)]dM(y)$

If in addition, F is non-lattice, then prove that

$$\lim_{t\to\infty} \Pr\{Y(t) \leq x\} = \frac{1}{\mu}\int_0^x [1 - F(y)]dy.$$

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